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Конечно-элементное моделирование поведения гранулированных материалов при малоцикловом нагружении

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Ключевые слова: модель состояния, конституционная модель, модель Мора-Кулона, модифицированная модель Кэм-Клэй, модифицированная модель Друкера-Прагера, гистерезис, малоцикловое нагружение

В работе рассмотрено поведение гранулированного материала при малоцикловом нагружении. В ходе моделирования использовались несколько упруго-пластических конституционных моделей: модель Мора-Кулона, модифицированная модель Кэм-Клэй и модифицированная модель Друкера-Прагера. Поведение материала анализировано применительно к форме гистерезиса на диаграмме сигма-эпсилон. Моделирование осуществлено в рамках квазистатического расчета с инфинитезимальными деформациями. Также рассмотрены энергетические потери в ходе гистерезисного поведения.

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Finite Element Simulation of Behaviour of Granular Materials under Low-Cyclic Loading

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Different continuous elastic-plastic constitutive models (Mohr-Coulomb model, modified Cam-Clay model, modified Drucker-Prager model) are analysed with respect to their ability to simulate hysteretic behaviour of granular materials under the low-cyclic kinematic loading. The analysis is done in the framework of the quasi-static infinitesimal deformations. In case of the Mohr-Coulomb constitutive model, hysteresis degeneration into the straight line is revealed. Energy loss due to the hysteretic behaviour is studied. It is shown that horizontal areas representing the elastic behaviour are significantly longer for the Mohr-Coulomb criterion.

Keywords: constitutive models, elasto-plasticity, Mohr-Coulomb model, modified Cam-Clay model, modi-fied Drucker-Prager model, hysteretic behaviour, cyclic loading

Introduction

A simulation of nonlinear deformation processes of various geological materials is widely used in civil engineering and petroleum industries [1-4]. In general, accurate modelling requires several components: use of the appropriate constitutive model, formulation of the well-posed boundary-value problem, and correct interpretation of the obtained results. The choice of the right constitutive model is crucial for the proper account of the mechanical behaviour of granular materials (such as soils, powder materials, or even pharmaceutical compounds). The present analysis comprises several constitutive models used for simulating behaviour of elastic-plastic materials under low-cycle loadings and comparing the corresponding stress-strain diagrams (i.e. hysteresis loops) and energy dissipations during elastic-plastic deformations.

1. The Mohr-Coulomb constitutive model

The Mohr-Coulomb yield criterion is a widely used classical model of plastic deformation of soils. Approaches based on Mohr-Coulomb model employ either theory of plasticity without hardening or plasticity with the isotropic hardening (i.e. when yield surface expands uniformly).

The Mohr-Coulomb criterion represents linear relationship between shear and normal stresses. This dependency has the following form

$$\tau = c + \sigma \tan(\varphi) \tag{1}$$

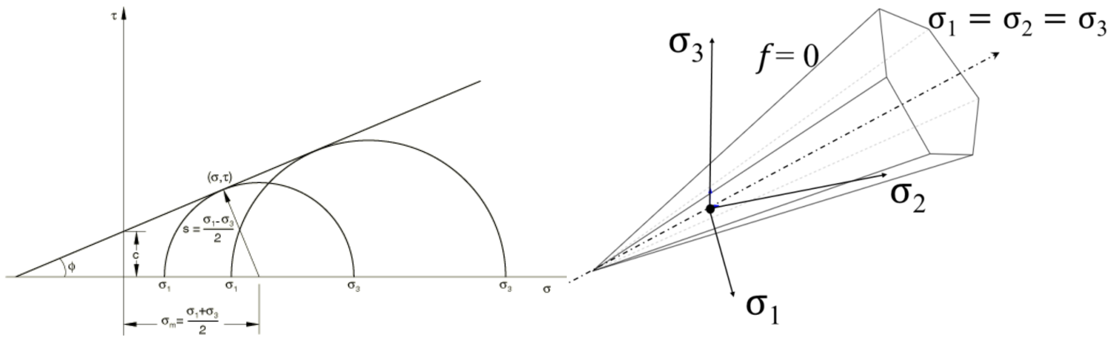

Fig. 1. Mohr-Coulomb yield surface: a) in the space of shear and normal stresses; b) in principal stress space [5]

Hereinafter, for describing the loading process the diagram $q-p$ is used, where p is a measure of volumetric stress component

$$p=\frac{1}{3}\text{trace}(\sigma), \tag{2}$$

q is the Mises equivalent stress (deviatoric stress component)

$$q=\sqrt{\frac{3}{2}\left(\mathbf{d}_\sigma:\mathbf{d}_\sigma\right)} \tag{3}$$

Finite element analysis of material behaviour is carried out in the following way: three faces of a cube, representing a single finite element, are loaded kinematically, whereas the remained faces are constrained from normal displacements (Fig. 2). For the sake of convenience, the subjected loading is implemented in such a way that its volumetric component remains constant during the test.

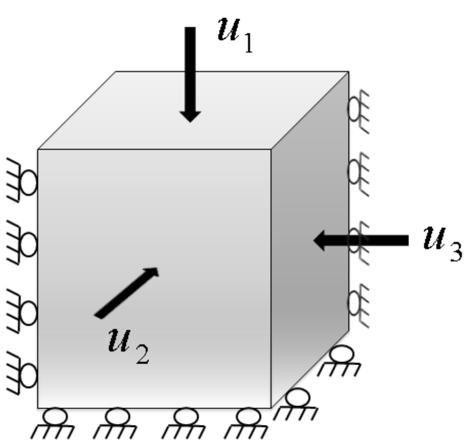

Fig.2. Boundary conditions of the single cube

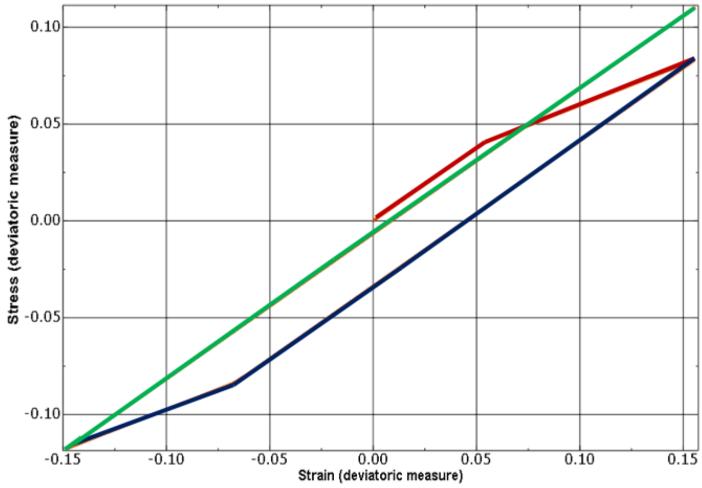

Fig.3. Stress-strain diagram (deviatoric components) according to the Mohr-Coulomb model

Here and further, the cyclic loading is considered: first of all, the compaction towards three faces is executed (red colour in Fig. 3), then loading changes its sign (blue colour). At last, the compaction is repeated (green colour).

Concerning the red section of the diagram, a slope change is associated with the hardening initiation. In other terms, as soon as the stress state 'touches' the initial yield surface the material properties should be changed, thus expanding the yield locus. Analysis of the blue section (loading with the opposite sign) reveals that hardening takes place noticeably later. Further, after subsequent change of loading sign (green colour) hardening does not occur at all, that is hysteresis degenerates into the straight line at the second cycle. This fact implies that the Mohr-Coulomb model disagrees with cyclic loading modelling since sooner or later deformations become elastic. Apparently, it is caused by the hardening type which is specified as isotropic in the approaches based on the Mohr-Coulomb model.

2. The modified Cam-Clay critical state model

In this section, the modified Cam-Clay (MCC) model [6] is considered. The MCC model belongs to cap plasticity constitutive models and has a number of undeniable advantages. As such, it allows more realistic modelling of volume changes, ease of use in finite element modelling, and accounting the hardening and softening effects depending on the stress state.

Yield surface of the MCC model is an ellipsoid (Fig. 4) described by the equation

$$\frac{q^2}{p^2} + M^2\left(1 - \frac{p_0}{p}\right) = 0 \qquad (4)$$

where M is the slope of the critical state line (CSL) $q = Mp$, p_0 is the preconsolidation pressure. It is worth noting that the CSL intersects the yield surface at the point of maximum Mises equivalent stress q.

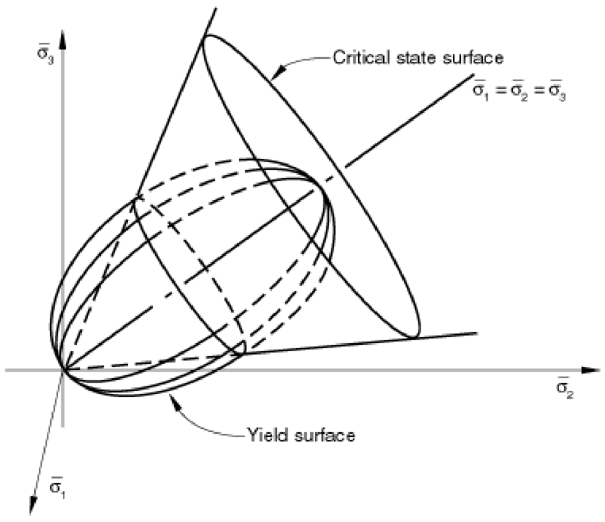

Fig. 4. Yield surface of the modified Cam-Clay model (ellipsoid) and critical surface (cone) [5]

Input parameters of the MCC model:

Set up of the model requires five material parameters, namely:

1. λ - the slope of the normal compression line (NCL) (Fig. 5.a);
2. κ - the slope of the unloading-reloading line in the $v - \ln(p)$ space (Fig. 5.b);
3. M - the slope of the CSL in the $q - p$ space;
4. N - the specific volume under the unit pressure calculated according to the NCL

   or

   Γ - the specific volume under the unit pressure calculated according to the CSL;
5. μ - Poisson ratio

   or

   G - the shear modulus.

Also it is necessary to specify the initial consolidation level. This may be done by specifying

p_0 - the preconsolidation pressure

or

OCR – the overconsolidation ratio $\text{OCR} = \dfrac{p_0}{p}$ (ratio of the preconsolidation pressure to the current pressure).

As it has been said before, one of the advatages of the MCC model is the account of hardening and softening behaviour. Under shearing, the material behaves elastically until the current stress state hits the yield surface. From then on depending on where the yield surface was reached, it starts to grow or to shrink.

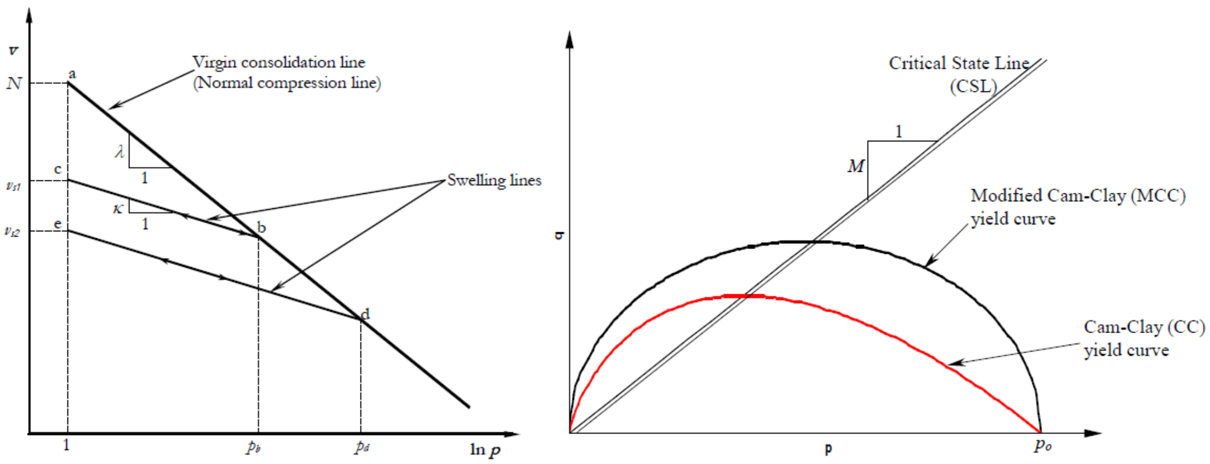

Fig. 5. Material behaviour in the modified Cam-Clay model: a) the NCL and unloading-reloading lines; b) yield curve in the $q - p$ space [7]

It is shown below that if yielding occurs to the right of the intersection of CSL with the yield curve, the material behaviour is characterised by hardening and compaction (Fig. 6.b). In this figure the yield surfaces at the initial and critical states are marked with black and red colours respectively, and two arbitrary positions of the surface during hardening are marked with blue.

If the yielding occurs to the left of the intersection between the CSL and the yield curve, the material behaviour is characterised by softening and dilatancy (Fig. 6.a). As seen from the diagram, the size of the initial envelope decreases.

It is worth noting that when the MCC model is used, the nonzero initial stress state must be specified. The MCC model is not valid in case of negative initial mean stress (i.e. according to soil mechanics, the sample is in extension state).

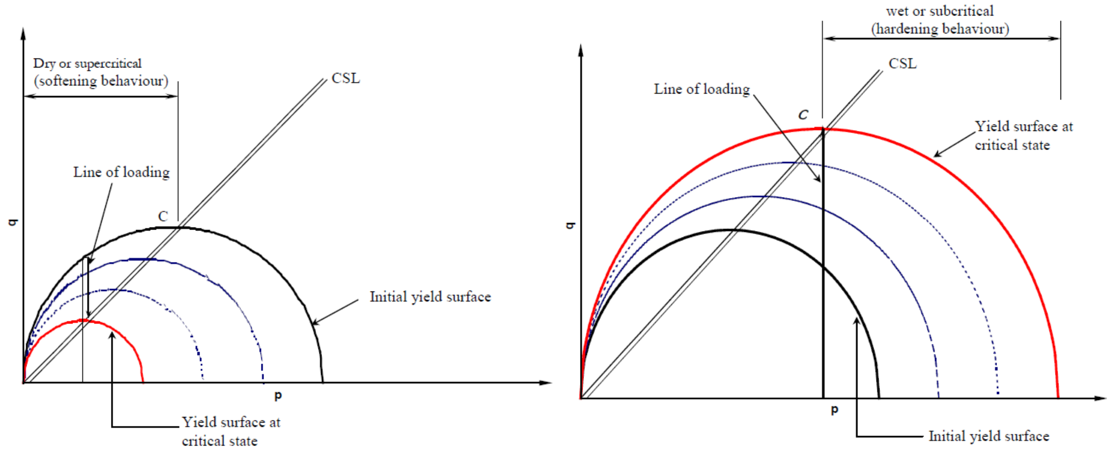

Fig. 6. Evolution of the yield envelope in the MCC model: a) softening in the «supercritical zone» zone; b) hardening in the «subcritical» zone [7]

As it was already made with the Mohr-Coulomb model, the loading starts with compaction, then unloading and expansion begin, then compaction is repeated. Fig. 7.a depicts the loading regime in terms of mean stress and volumetric strain.

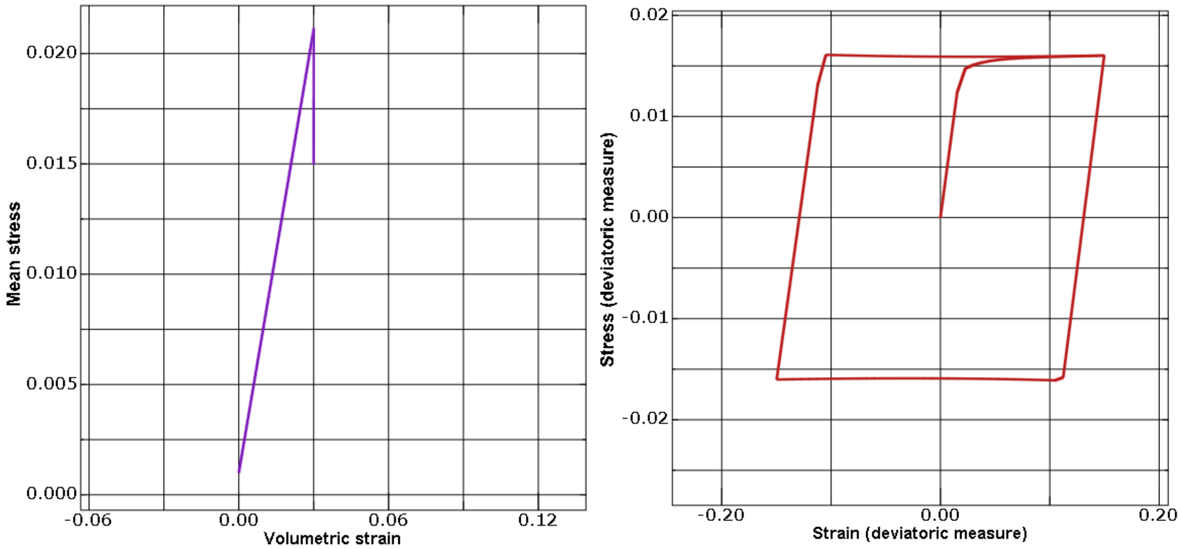

Fig. 7. Material exhibits softening behaviour: a) stress-strain diagram in terms of mean stress and volumetric strain; b) deviatoric components of the stress-strain diagram

As observed from Fig. 7.b, hysteresis plotted in deviatoric space simply represents the elastic-plastic behaviour of the material and reveals neither hardening nor softening effect. This fact can be explained by the opposite signs of maximal and minimal loads subjected to different faces, so calculation of the deviatoric component gives the constant value. For indicating hardening and softening behaviours, the stress-strain diagrams are plotted in terms of principal stresses and strains (Fig. 8).

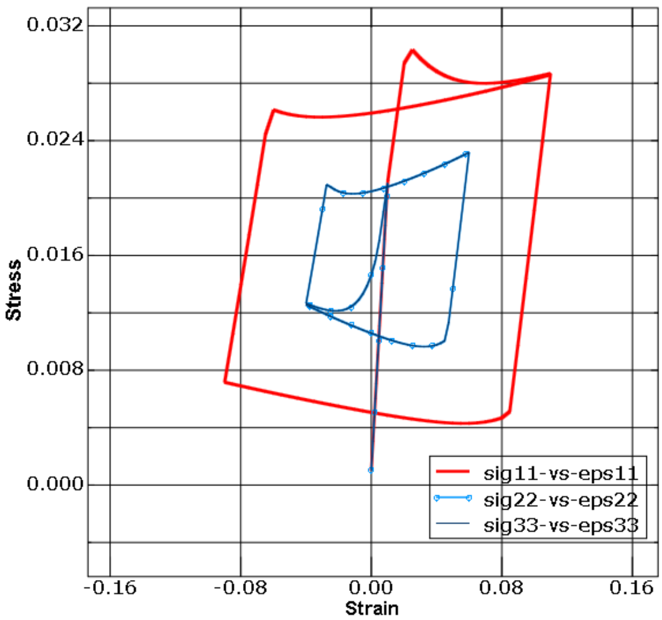

Fig. 8. Stress-strain diagrams for the principal stresses and strains revealing softening effect

After analysing softening behaviour beyond the stress peak (i.e. yielding occurred in the supercritical zone), it is logical to investigate hardening. The corresponding load should provide the yield curve is reached in the subcritical zone. So, volumetric component of the kinematic loading has been modified for the subcritical case.

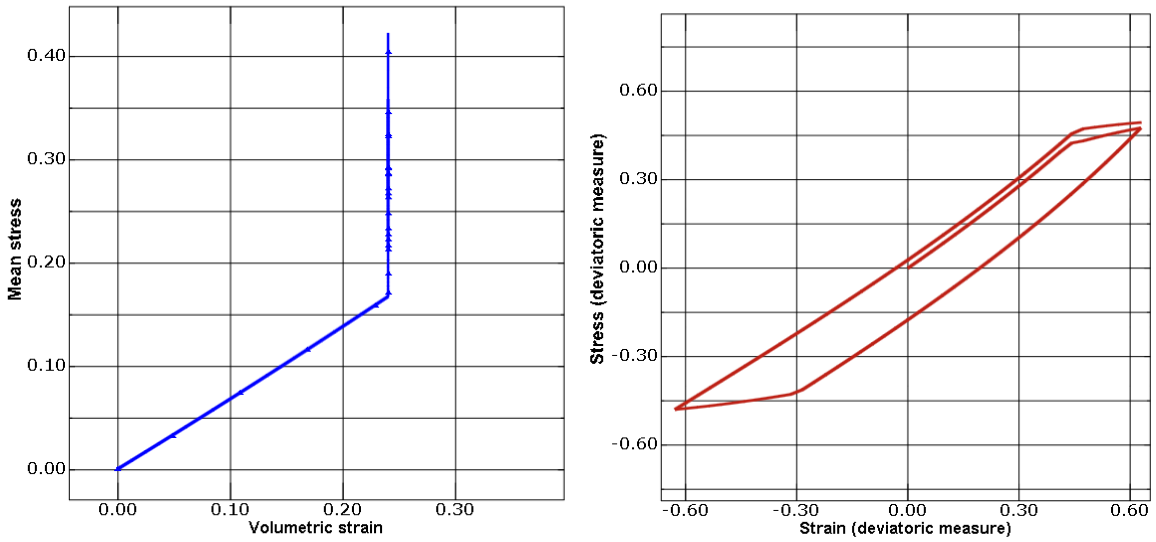

Fig. 9. Material exhibits hardening behaviour: a) stress-strain diagram in terms of mean stress and volumetric strain; b) deviatoric components of the stress-strain diagram

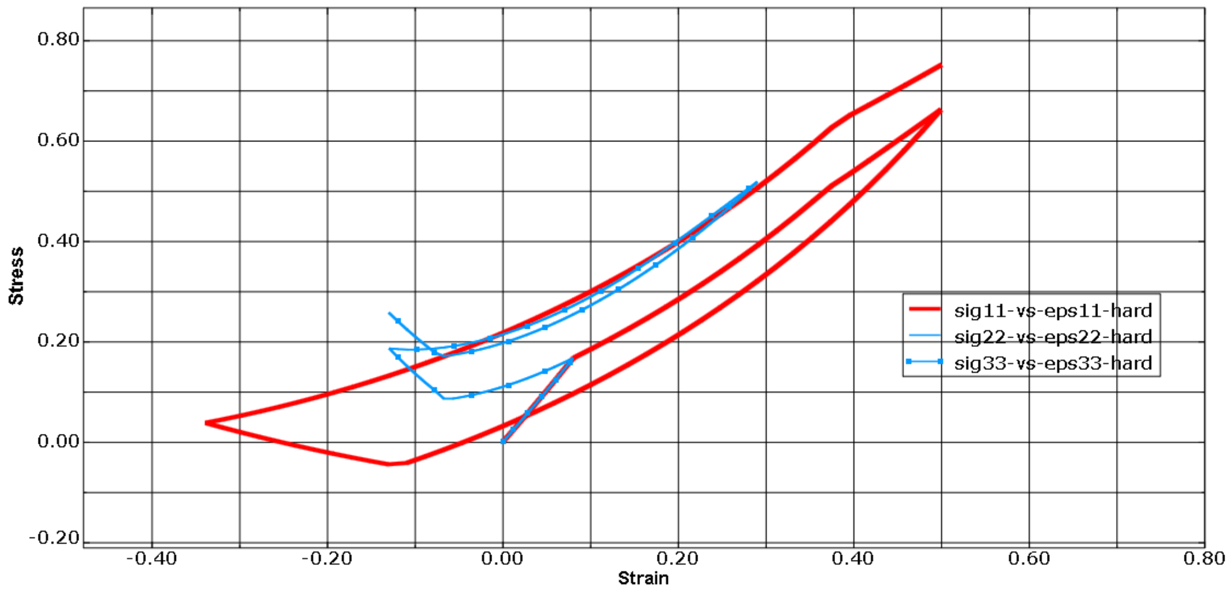

Fig. 10. Stress-strain diagrams for the principal stresses and strains revealing the hardening effect

3. The modified Drucker-Prager constitutive model

The modified Drucker-Prager plasticity model is mostly intended for geological materials that exhibit pressure-dependent yield [8]. The yield surface comprises the two main parts: a shear failure surface, providing predominantly shearing flow, and a cap, which intersects the mean stress axis (Fig. 11).

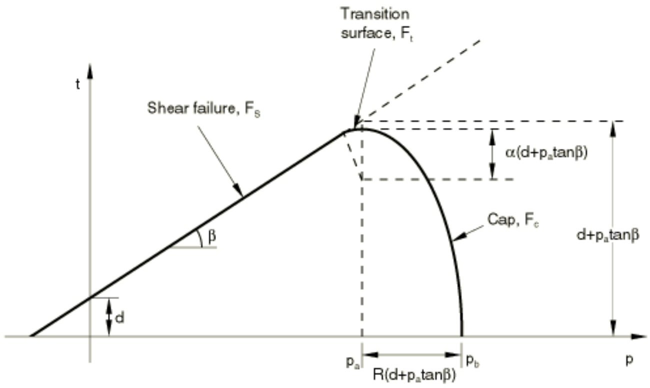

Fig. 11. Yield envelope of the modified Drucker-Prager model [1]

The cap plays two roles: it bounds the yield surface in hydrostatic compression, thus providing an inelastic hardening mechanism, and it controls the volumetric dilatancy when the material yields in shear, thus providing a softening mechanism.

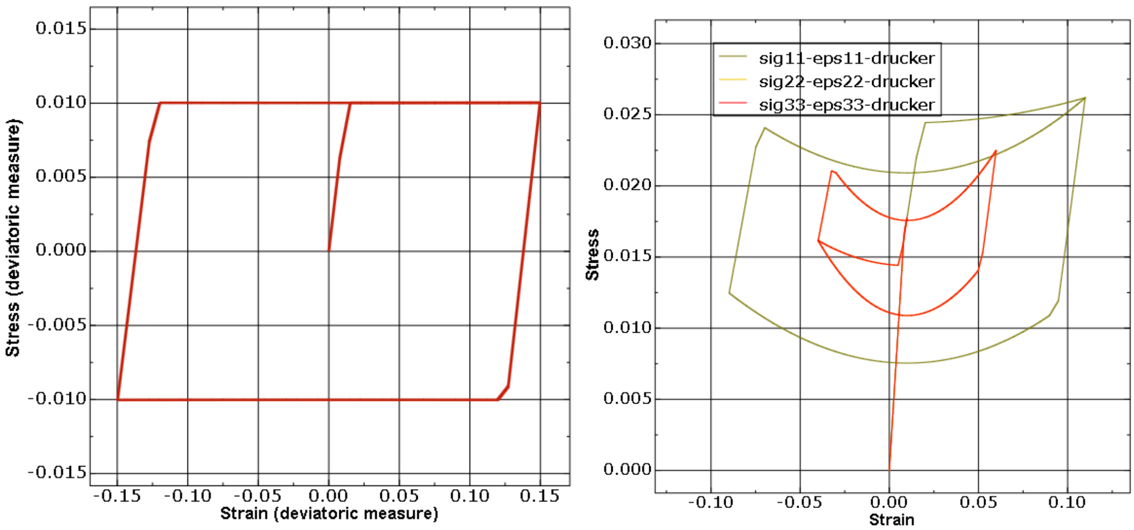

Fig. 12. Stress-strain diagrams for the modified Drucker-Prager model: Material exhibits hardening behaviour: a) for the deviatoric components of the stress-strain diagram; b) for the principal stresses and strains

As in the case of the MCC model, in deviatoric components the yield threshold is independent of plastic deformation, so it represents the case of perfect elasto-plasticity (Fig. 12.a).

For capturing the hardening effect, the stress-strain diagrams are plotted in terms of principal stresses and strains (Fig. 12.b).

Energy loss due to the hysteretic behaviour

Following the study of the presented constitutive models, it becomes interesting to analyse the energy loss during the deformation. The increase of plastic dissipation (Fig. 13) characterises the hysteresis efficiency: higher values correspond to more intense dissipation over the deformation cycle, and horizontal areas represent elastic deformation. In this context, it is imperative to note that horizontal areas are significantly longer for Mohr-Coulomb criterion, and the over last cycle there is no increase in energy dissipation. The latter fact implies that Mohr-Coulomb model does not allow to provide the accurate simulation of cyclic loadings.

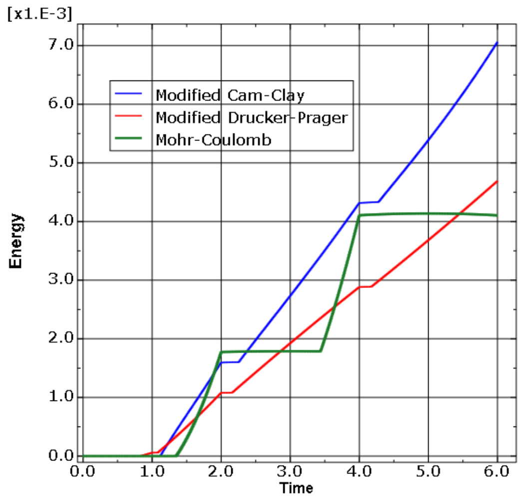

Fig. 13. Energy increase while the material is being deformed

Conclusion

According to the presented analysis of several constitutive models, it becomes possible to choose the most appropriate constitutive model allowing correct representation of the material behaviour. As it was shown, when it comes to cyclic loadings, the Mohr-Coulomb constitutive model (and classical Drucker-Prager model as well) is not able to describe the appropriate material behaviour with non-degenerate hysteretic loops. However, both these models can be used if hardening parameters are not specified: in such a case no degradation of the hysteretic behaviour occurs. At the same time, the modified Cam clay and modified Drucker-Prager models accurately describe material behaviour under the cyclic loading without additional restriction on non-hardening response.

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